OPTICAL BISTABILITY IN A NONLINEAR FABRY-PEROT INTERFEROMETER : A SIMULATION BASED ON THE BEAM PROPAGATION METHOD

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Résumé: La théorie de la bistabilité dans un Fabry-Pérot de type Kerr, est reprise dans le cas d'illumination en ondes planes de polarisation TE, et à incidence normale. Ensuite, nous proposons une nouvelle technique de simulation, basée sur la méthode dite 'Beam Propagation method'. Les premiers résultats de cette simulation sont présentés.
Abstract: The classical description of optical bistability behaviour of a nonlinear Kerr-type Fabry-Perot etalon, is given for normal incidence in the case of TE-plane waves. Thereafter, we propose a new simulation method of the Fabry-Pérot, based on the so-called 'Beam Propagation Method'. First numerical results are reported.

1. Introduction

In the last decade, optical bistability studies have known a tremendous development, due to the possibilities it offers to optical computing and information processing. Indeed, optical bistable devices have properties which are similar to those of electronic devices, and make possible the realization of optical memories, switches, amplifiers and logic elements. Moreover, optical devices are well adapted for parallel processing, so that one can expect dramatic improvements of the performances of future computers, in a.o. hybrid optoelectronic devices or all-optical computers [1].
We recall in section 2 the plane-wave theory developed by Marburger and Felber [2], and by Miller [3]. In section 3 we propose a simulation based on the well-known beam propagation method [13], which is able to describe the influence of other factors, like diffraction and wave-shape effects.

2. Description of the nonlinear Fabry-Pérot in the case of plane waves with normally incidence.

Consider an isotropic Fabry-Pérot filled with a Kerr-type medium with weak linear absorption. This medium is characterized by a third-order susceptibility proportional to the total cavity electric field E:

$$\chi(\omega, r) = \chi_L + i\,\chi_L' + \chi^{(3)}\,|E(\omega,r)|^2 \tag{2.1}$$

The cavity has thickness D and front and back-face reflectivities, R_F and R_B. Note that, except for thin films, the nonlinear effects can always be neglected at the boundary, so that $R_{F,B}$ can be considered as a constant [5].
Suppose a monochromatic TE-plane wave of wave vector $\mathbf{k_0}$ impinging perpendicularly onto the cavity. Its propagation can be described by the Maxwell equations, which, in the steady-state, and after extraction of the time-dependence, can be reduced to the scalar nonlinear Helmoltz equation :

$$\Delta^2 E + k_0^2\,(1 + \chi_L + i\,\chi_L' + \chi^{(3)}|E|^2)\,E = 0 \tag{2.2}$$

Inside the cavity, E can be evaluated in terms of two global fields corresponding to the 2 propagation directions:

$$E(z) = E_F(z) + E_B(z) \quad \text{with} \quad E_F(z) = \mathcal{E}_F(z)\, e^{i(kz + \Phi_F(z))} \;\; ; \;\; E_B(z) = \mathcal{E}_B(z)\, e^{i(-kz + \Phi_B(z))} \qquad (2.3);\ (2.4);\ (2.5)$$

where $k=k_0n_0$ and $\Phi_{F,B}$ describe the nonlinear contribution to the phase. Substituting those expressions in (2.2) and taking the slowly varying approximation into account (variations of the second order in $\mathcal{E}_{F,B}$ and $\Phi_{F,B}$ are neglected) leads, after averaging over some spatial periods, to a new set of equations:

$$\partial_z \Phi_F = \frac{k_0^2\chi^{(3)}}{2k}\,[\,\mathcal{E}_F^2(z) + 2\mathcal{E}_B^2(z)] \;\; ; \;\; \partial_z \Phi_B = -\,\frac{k_0^2\chi^{(3)}}{2k}\,[\,2\mathcal{E}_F^2(z) + \mathcal{E}_B^2(z)] \qquad (2.6)\ ;\ (2.7)$$

$$\partial_z \mathcal{E}_F = -\,\frac{k_0^2\,\chi_L'}{2k}\,\mathcal{E}_F \;\; ; \;\; \partial_z \mathcal{E}_B = \frac{k_0^2\,\chi_L'}{2k}\,\mathcal{E}_B \qquad (2.8)\ ;\ (2.9)$$

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Defining the intensity absorption as $\alpha = k_0^2 \, \chi'_L / 2k$, and using the proper boundary conditions result in:

$$E_F(z) = \mathcal{E}_F(0)\, e^{-\frac{\alpha}{2} z}\, e^{ikz}\, e^{i\Phi_F(z)} \quad ; \quad E_B(z) = \mathcal{E}_B(D)\, e^{-\frac{\alpha}{2}(D-z)}\, e^{-ikz}\, e^{i\Phi_B(z)} \qquad (2.10) ; (2.11)$$

The half-roundtrip nonlinear phase change is obtained by integrating the equations (2.6) and (2.7):

$$\Delta\Phi_{NL} = \int_0^D [\partial_z \Phi_F - \partial_z \Phi_B]\, dz \qquad (2.12)$$

It is useful to define an effective mean intensity I_{eff}, in order to describe the phase change. Following (2.6, 7, 12), the total roundtrip phase becomes:

$$\Delta\Phi = k_0 n_0 D + k_0 n_2 I_{eff} = k_0 n_0 D + \frac{3}{2} \frac{k_0^2 \chi^{(3)}}{2k} \int_0^D [\mathcal{E}_F^2(z) + \mathcal{E}_B^2(z)] dz \qquad (2.13)$$

where I_{eff} is defined as $I_{eff} = (\varepsilon_0 n_0 c / 2) \int_0^D [\mathcal{E}_F^2(z) + \mathcal{E}_B^2(z)] dz$ with $n_2 = 3/2\, \chi^{(3)} / n_0^2 \varepsilon_0 c$. (2.14)

The overall response of the Fabry-Pérot can now be obtained from the boundary conditions (continuity of the electromagnetic field and of its normal derivative). They yield a set four algebraic equations, which can easily be solved. They lead to:

1) The overall transmission τ

$$\tau = \frac{I_T}{I_0} = \frac{(1-R_B)(1-R_F)e^{-\alpha D}}{(1-R_{eff})^2} \cdot \frac{1}{1 + F \sin^2(\Delta\Phi/2)} \qquad (2.15)$$

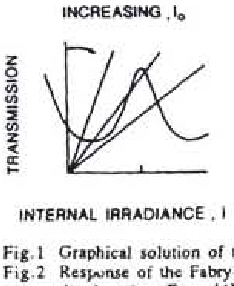
where

$$R_{eff} = \sqrt{R_F R_B}\, e^{-\alpha D} \quad ; \quad F = \frac{4\, R_{eff}}{(1-R_{eff})^2} \qquad (2.16) ; (2.17)$$

2) A relation between the effective cavity intensity and the transmitted intensity I_T.

$$I_{eff} = \frac{1}{\alpha D} \frac{(1-e^{-\alpha D})\,(1+R_B e^{-\alpha D})}{(1-R_B)\, e^{-\alpha D}} I_T \qquad (2.18)$$

One recognizes, in (2.15) a more general form of the so-called Airy formula which characterizes the linear Fabry-Pérot response [6]. Here the phase change is given by (2.13).
So we have obtained a set of 3 equations (2.13), (2.15), (2.18) which provides a complete description of the Fabry-Pérot cavity in terms of I_0, I_{eff}, I_T, with as cavity parameters, $R_{F,B}$ and αD. A graphical solution of this set is shown in fig. 1 , and the response of the cavity can be evaluated by plotting the equivalent curve $I_T = f(I_0)$ (see fig. 2).



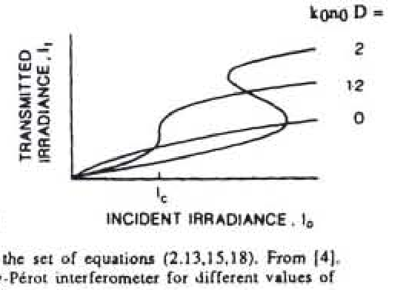


Fig.1 Graphical solution of the set of equations (2.13,15,18). From [4].
Fig.2 Response of the Fabry-Pérot interferometer for different values of the detuning. From [4].

Figure 1 and 2 show that for small values of I_0 there is only one solution. If I_0 grows, the slope of the straight line diminishes, and from a certain value on, it has 3 common points with the Airy curve. It can be shown that only 2 of these solutions correspond to stable states. This is optical bistability. If I_0 grows further, the system has more solutions which corresponds to the optical multistable region.
Critical intensities and detunings giving the treshold to optical bistability are important for cavity optimization and have been studied elsewhere [3].

3. Description of a real Fabry-Pérot interferometer.

Millers' model explained in section II is an ideal one. In fact, the experience shows that many factors influence the response of a real Fabry-Pérot etalon [4], like a.o. oblique incidence, polarization, diffractive effects, nonlinearities due to thermal effects, higher-order electronic nonlinearities, etc. Furthermore, the practical realization of arrays of memory or logic elements require the use of many very thin beams, and such situations are impossible to describe analytically. Therefore, it is important to develop simulation methods which take those effects into account, and which are able to treat the case of (a few) limited beams. Some work has already been reported in this direction [7, 8, 9, 10]. Here we propose a simulation scheme based on the well-known Beam Propagation Method. To the best of our knowledge, the nonlinear Fabry-Pérot has not been approached numerically by this method which yields fruitful results in linear integrated optics. It is the purpose of this paper to show that the BPM can be extended to the present problem.

A. The Beam Propagation Method (BPM).

The BPM is a very elegant method which allows to describe, by means of very simple basic algorithms, the propagation of a large range of beam profiles in rather complex structures [11, 12, 13]. This method is an iterative one: it consists of calculating the amplitude of the electric field after a very small propagation step Δz, by means of the scalar inhomogeneous Helmoltz equation and proper boundary conditions:

$$\Delta^2 E + k_0^2 n^2(x,z)\ E = 0 \tag{3.1}$$

Due to the smallness of Δz, simplifications occur, and a solution can easily be found. The same calculation is repeated until z reaches the final value z=D corresponding to the output plane.
The BPM requires the following conditions: The refractive index of the medium must be continuous and characterized by small variations Δn around a reference value n_0. In its classical form the BPM is concerned with beams which propagate without reflection and satisfy the paraxial approximation.
The treatment occurs in two steps. First, the propagation is determined by neglecting the refractive index variations: this is easily performed by the use of the Fourier transformation in the x-direction (FT). Afterwards, the amplitude is calculated by taking into account the index variation as a small perturbation. Due to the small value of Δn and Δz, the correction takes the form of a simple phase factor. So the solution of (3.1) can finally be written in a very simple way:

$$E(x,z_0+\Delta z) = e^{i k_0 \Delta n \Delta z}\ FT^{-1}\ \{ e^{i k_z \Delta z}\ FT\ (E(x,z_0)) \} \tag{3.2}$$

where k_z is the z-compound of $k_0 n_0$ of the corresponding spatial frequency in the FT. The use of very efficient FFT algorithms makes this method very attractive for the description of complex amplitude and refractive index profiles.

B. Simulation of the propagation in a Fabry-Pérot etalon using BPM.

First of all, one verifies that this problem obeys the validity conditions of applicability of the BPM: the refractive index variations result from a third order polarization term which its proportional to the intensity: they are thus continuous and small with regard to the linear part of the refractive index. Further, due to the high reflectivity of the Fabry-Pérot cavity, the paraxial approximation is always verified, even for high values of the angle of incidence.
The difficulty to use the BPM in the present problem is that only the incident beam is known, and not the internal field which is build up in the cavity, and which determines the local refractive index. In order to solve this problem, one uses successive approximations. First, the propagation of the incident beam after transmission in the nonlinear medium is described by BPM: the corresponding intensity is used to calculate the nonlinear refractive index at each point. After the calculation of the reflected beam, one disposes of a first evaluation of E_F and E_B at the back face. They are used as new boundary conditions at z=D for the Helmoltz equation, which is again solved with BPM, by describing simultaneously the propagation of both E_F and E_B to the input plane. So, we dispose at each point of a rather satisfactory value of the refractive

index, by adding the contributions of E_F and E_B to the intensity. Afterwards, the reflection of E_B on the input face, added to the transmitted part of the incident beam, gives a new and better E_F simultaneously. After a few successive approximations, the calculated electric field reaches the real stationary value in the cavity. It is then easy to calculate the total transmitted field, and the transmission coefficient of the etalon.
Now, the BPM must be adapted to the simultaneous description of both the forward and backward beams. First one considers the "unperturbed wave equation":

$$\Delta^2 E^0 + k_0^2 \left(1 + \chi_L + i\,\chi_L' \right) E^0 = 0 \tag{3.3}$$

Using the Fourier transform in the x-direction of (2.3) leads easily to the solution

$$E^0(x,z_0+\Delta z) = FT^{-1}\left\{ e^{i k_z' \Delta z}\, FT\left(E_F(x,z_0) \right) + e^{-i k_z' \Delta z}\, FT\left(E_B(x,z_0) \right) \right\} = E_F^0(x,z_0+\Delta z) + E_B^0(x,z_0+\Delta z) \tag{3.4}$$

where $k_z' = k_z + i\, k_0 n_0 \alpha / 2k_z$. This is used to solve (2.2) by searching a solution in the form:

$$E(x,z) = E_F^0(x,z)\, e^{\Gamma_F(x,z)} + E_B^0(x,z)\, e^{\Gamma_B(x,z)} \tag{3.5}$$

Writing ΔE and neglecting the second order variation of $\Gamma_{F,B}$ and using (3.3) leads to

$$[2\, \partial_x E_F^0\, \partial_x \Gamma_F + 2\, \partial_z E_F^0\, \partial_z \Gamma_F]\, e^{\Gamma_F} + [2\, \partial_x E_B^0\, \partial_x \Gamma_B + 2\, \partial_z E_B^0\, \partial_z \Gamma_B]\, e^{\Gamma_B} + k_0^2 \chi^{(3)} |E|^2 E = 0 \tag{3.6}$$

A priori, there is no relationship between the phases of the field compounds and their derivatives, so that it is not possible to split this equation to find expressions for $\Gamma_{F,B}$ separately. However, it is known that the refractive index of the cavity is high, so that the angular spectrum of the field within the Fabry-Pérot is very narrow, with a mean frequency K_x corresponding to the value K_z following the z-direction. Rewriting E_F and it derivative as:

$$E_F^0(x,z) = e^{i K_z \Delta z} \int_{-\infty}^{+\infty} \mathcal{E}_F^0(k_x)\, e^{i (k_z' - K_z) \Delta z}\, dk_x \tag{3.7}$$

$$\partial_z E_F^0 = i\, K_z\, E_F^0 + e^{i K_z \Delta z} \int_{-\infty}^{+\infty} \mathcal{E}_F^0(k_x)\, i\,(k_z' - K_z)\, e^{i (k_z' - K_z) \Delta z}\, dk_x \tag{3.8}$$

it can be seen that, for a high refractive index, $K_z \gg | k' - K_z |$, so that $\partial_z E_F$ can be assimilated to $i\, K_z\, E_F$. The same reasoning is valuable for E_B, and (3.6) can thus be split into the contributions in E_F and E_B.
The perturbations $\Gamma_{F,B}$ can be developed in series of Δz and limited to the first terms:

$$\Gamma_F = \Gamma_{1F}(x)\, \Delta z + \Gamma_{2F}(x)\, \Delta z^2 + \dots \quad ; \quad \Gamma_B = \Gamma_{1B}(x)\, \Delta z + \Gamma_{2B}(x)\, \Delta z^2 + \dots \tag{3.9; 3.10}$$

Replacing (3.9,10) into (3.6) gives an new series in Δz, with an independent term which leads to:

$$\Gamma_{1F} = -\frac{k_0^2 \chi^{(3)} [|E_F|^2 + 2|E_B^2|]}{2 \partial_z E_F^0} E_F^0 \; ; \; \Gamma_{1B} = -\frac{k_0^2 \chi^{(3)} [2|E_F|^2 + |E_B^2|]}{2 \partial_z E_B^0} E_B^0 \tag{3.11; 3.12}$$

The examination of the term in Δz of (3.6), shows that the higher-order corrections of (3.9,10) are negligible. So we have obtained a rather simple solution of the Helmoltz equation, given by (3.4,5,11,12). This method has been implemented and successfully compared, in the case of plane waves, with a direct resolution method of (2.13,15,18), as shown in fig. 3.
Fig. 4 shows several response curves for plane wave situations, corresponding to different values of the detuning. A typical refractive index profile in the cavity is represented in fig.5 as a function of z and E_0. It shows that the influence of the field is very weak for a low incident field. At the switch to a higher stable state, represented by the discontinuity of the surface, the cavity intensity becomes much more important, and the interferences are much more constructive, leading the system to a situation close to resonance, as being shown on fig. 1.

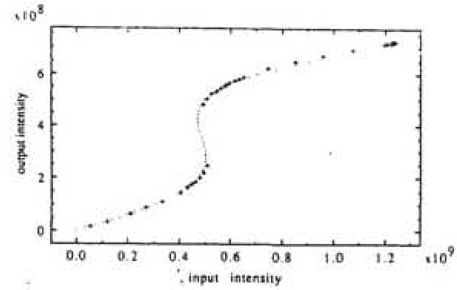


Fig. 3 Comparison between simulation results and analytical data [5]. ..... : analytical results; +: simulation points. $D=5\mu m$, $\alpha=1000\ m^{-1}$; $n_0=3.56$; $\chi^{(3)}=10^{-12}\ m^2/V^2$; $\lambda=1.06\mu m$

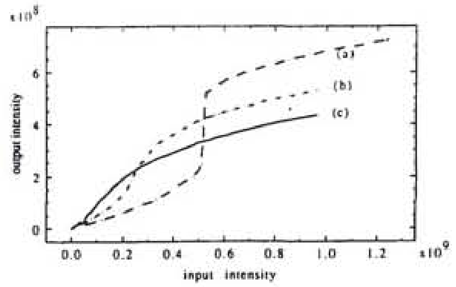


Fig.4 Response for different values of the thickness: (a)$D=5\ \mu m$; (b)$D=5.026\ \mu m$; (c)$D=5.043\mu m$. $\alpha=2000\ m^{-1}$; $n_0=3.56$; $\chi^{(3)}=10^{-12}\ m^2/V^2$; $\lambda=1.06\mu m$

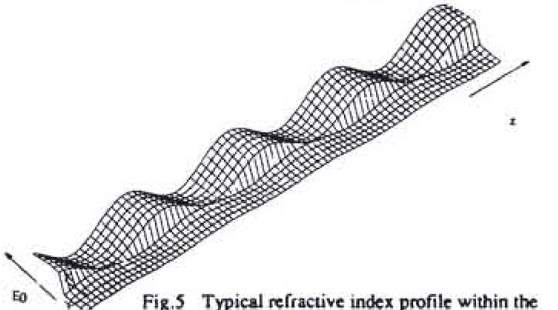


Fig.5 Typical refractive index profile within the cavity, as a function of z and E_0

Conclusion

After explaining the plane-wave model of the propagation in a Fabry-Pérot interferometer, we mentioned the necessity to develop new simulation methods for the description of effects like diffusion, diffraction, or the influence of the beam shape on the Fabry-Pérot response. We proposed a simulation scheme based on the BPM, which provides a rather simple description of the propagation in the cavity, in theory for any wave profile (so far as the process converges), and taking into account the diffraction effects. So far, the results obtained in the case of plane waves are very encouraging, but this model will be improved by adding the treatment of diffusion, which is expected to have a stabilizing effect on the amplitude of the simulation, and to improve further the convergence process, and the efficiency of the method.

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